The appendix of "Fair Clustering Ensemble with Equal Cluster Capacity"

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1 Appendix A: Experimental results of ensemble of base results with different cluster numbers

In our setting, we assume that the cluster numbers of all base results are the same. However, our method can be extended to handle base results with different cluster numbers. In this case, we should modify the rotation matrices $\mathbf{R}^{(i)}$. For example, if the cluster number of the i-th base result is c_i, we should modify $\mathbf{R}^{(i)}$ as $\mathbf{R}^{(i)} \in \mathbb{R}^{c_i \times c}$. The rest parts are similar. We conduct a group of experiments to evaluate it. In this experiment, we generate 10 base clustering results, whose cluster numbers are randomly selected in the range $[2, \sqrt{n}]$, where n is the number of instances. The experimental results are shown in Tables 1, 2, and 3. PFREFF [1] cannot handle this case and thus it has no results. The results show that our method can outperform other methods regarding fairness and cluster capacity equality. When considering the ACC and NMI, our method FCE or the degenerated FCE-f is comparable with other methods. This result is similar to the results reported in the main body of this paper.

References

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TABLE 1: Experimental results with different base clusterings on MNIST-USPS and Reverse-MNIST data sets. The best and second best results are denoted in **bold** and underlined, respectively. FCE-f represents the degenerated version of FCE without the fairness regularized term.

Methods	MNIST-USPS							Reverse MNIST						
	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE
KM	0.303 ±0.038	0.320 ±0.091	0.000 ±0.000	0.000 ±0.000	0.038 ±0.037	0.867 ±0.083	0.000 ±0.000	0.262 ±0.063	0.355 ±0.076	0.000 ±0.000	0.000 ±0.000	0.258 ±0.159	0.975 ±0.018	0.000 ±0.000
BCE [2]	0.362 ±0.038	0.336 ±0.038	0.000 ±0.000	0.000 ±0.000	0.319 ±0.072	0.972 ±0.009	0.000 ±0.000	0.362 ±0.034	0.338 ±0.024	0.000 ±0.000	0.000 ±0.000	0.354 ±0.128	0.976 ±0.014	0.000 ±0.000
RCE [3]	0.359 ±0.023	0.352 ±0.021	0.000 ±0.000	0.000 ±0.000	0.083 ±0.095	0.878 ±0.031	0.000 ±0.000	0.342 ±0.020	0.395 ±0.031	0.000 ±0.000	0.000 ±0.000	0.145 ±0.135	0.942 ±0.013	0.000 ±0.000
LWGP [4]	0.334 ±0.019	0.340 ±0.034	0.000 ±0.000	0.000 ±0.000	0.070 ±0.041	0.863 ±0.056	0.000 ±0.000	0.372 ±0.014	0.434 ±0.015	0.000 ±0.000	0.000 ±0.000	0.291 ±0.058	0.962 ±0.013	0.000 ±0.000
LWEA [4]	0.363 ±0.043	0.344 ±0.047	0.000 ±0.000	0.000 ±0.000	0.054 ±0.026	0.849 ±0.043	0.000 ±0.000	**0.384** ±0.010	**0.440** ±0.016	0.000 ±0.000	0.000 ±0.000	0.263 ±0.069	0.962 ±0.014	0.000 ±0.000
DREC [5]	0.345 ±0.027	0.328 ±0.026	0.000 ±0.000	0.000 ±0.000	0.090 ±0.021	0.883 ±0.019	0.000 ±0.000	0.312 ±0.010	0.324 ±0.026	0.000 ±0.000	0.000 ±0.000	0.242 ±0.059	0.958 ±0.010	0.000 ±0.000
RSEC [6]	0.338 ±0.020	0.359 ±0.017	0.000 ±0.000	0.000 ±0.000	0.183 ±0.041	0.951 ±0.011	0.000 ±0.000	0.354 ±0.025	0.355 ±0.046	0.000 ±0.000	0.000 ±0.000	0.326 ±0.093	0.960 ±0.066	0.000 ±0.000
TRCE [7]	0.342 ±0.034	0.352 ±0.028	0.000 ±0.000	0.000 ±0.000	0.060 ±0.034	0.861 ±0.036	0.000 ±0.000	0.341 ±0.011	0.361 ±0.011	0.000 ±0.000	0.000 ±0.000	0.237 ±0.092	0.947 ±0.031	0.000 ±0.000
ECPCS-HC [8]	0.289 ±0.029	0.216 ±0.031	0.000 ±0.000	0.000 ±0.000	0.004 ±0.005	0.674 ±0.061	0.000 ±0.000	0.374 ±0.013	0.426 ±0.038	0.000 ±0.000	0.000 ±0.000	0.192 ±0.123	0.955 ±0.034	0.000 ±0.000
ECPCS-MC [8]	**0.365** ±0.027	0.361 ±0.023	0.000 ±0.000	0.000 ±0.000	0.087 ±0.036	0.895 ±0.025	0.000 ±0.000	0.375 ±0.011	0.438 ±0.019	0.000 ±0.000	0.000 ±0.000	0.339 ±0.052	0.977 ±0.005	0.000 ±0.000
CESHL [9]	0.245 ±0.054	0.168 ±0.094	0.000 ±0.000	0.000 ±0.000	0.006 ±0.010	0.507 ±0.246	0.000 ±0.000	0.321 ±0.018	0.367 ±0.039	0.000 ±0.000	0.000 ±0.000	0.168 ±0.137	0.914 ±0.067	0.000 ±0.000
FCE-f	**0.365** ±0.043	**0.362** ±0.042	0.000 ±0.000	0.000 ±0.000	0.023 ±0.014	0.836 ±0.053	0.000 ±0.000	0.366 ±0.022	0.412 ±0.048	0.000 ±0.000	0.000 ±0.000	0.131 ±0.060	0.916 ±0.056	0.000 ±0.000
FCE	0.342 ±0.028	0.321 ±0.018	**0.131** ±0.071	**0.479** ±0.195	**0.461** ±0.088	**0.980** ±0.029	**0.262** ±0.088	0.337 ±0.029	0.389 ±0.039	**0.160** ±0.050	**0.482** ±0.171	**0.485** ±0.072	**0.982** ±0.010	**0.401** ±0.035

TABLE 2: Experimental results with different base clusterings on D&S and HAR data sets. The best and second best results are denoted in **bold** and underlined, respectively. FCE-f represents the degenerated version of FCE without the fairness regularized term.

Methods	D&S							HAR						
	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE
KM	0.433 ±0.096	0.501 ±0.106	0.021 ±0.093	0.112 ±0.286	0.050 ±0.128	0.929 ±0.022	0.000 ±0.000	0.270 ±0.160	0.414 ±0.078	0.013 ±0.076	0.177 ±0.311	0.089 ±0.129	0.953 ±0.015	0.000 ±0.000
BCE [2]	0.506 ±0.037	0.513 ±0.025	0.000 ±0.000	0.343 ±0.255	0.169 ±0.090	0.965 ±0.018	0.000 ±0.000	0.536 ±0.110	0.443 ±0.140	0.003 ±0.005	0.913 ±0.064	0.507 ±0.137	0.982 ±0.010	0.000 ±0.000
RCE [3]	0.522 ±0.014	0.613 ±0.024	0.000 ±0.000	0.210 ±0.123	0.046 ±0.024	0.912 ±0.015	0.000 ±0.000	0.576 ±0.021	0.593 ±0.031	0.000 ±0.000	0.337 ±0.289	0.053 ±0.017	0.853 ±0.034	0.000 ±0.000
LWGP [4]	0.530 ±0.022	0.602 ±0.009	0.000 ±0.000	0.016 ±0.011	0.085 ±0.039	0.944 ±0.008	0.000 ±0.000	0.563 ±0.054	0.612 ±0.031	0.000 ±0.000	0.168 ±0.237	0.028 ±0.021	0.846 ±0.030	0.000 ±0.000
LWEA [4]	0.541 ±0.016	0.612 ±0.016	0.000 ±0.000	0.066 ±0.146	0.086 ±0.061	0.950 ±0.011	0.000 ±0.000	0.630 ±0.033	0.613 ±0.023	0.000 ±0.000	0.409 ±0.298	0.078 ±0.055	0.889 ±0.0118	0.000 ±0.000
DREC [5]	0.512 ±0.032	0.612 ±0.019	0.000 ±0.000	0.147 ±0.131	0.075 ±0.029	0.935 ±0.017	0.000 ±0.000	**0.632** ±0.071	**0.635** ±0.042	0.000 ±0.000	0.849 ±0.141	0.277 ±0.230	0.941 ±0.038	0.000 ±0.000
RSEC [6]	0.483 ±0.028	0.551 ±0.014	0.000 ±0.000	0.311 ±0.204	0.102 ±0.071	0.946 ±0.19	0.000 ±0.000	0.591 ±0.035	0.586 ±0.029	0.000 ±0.000	0.502 ±0.056	0.091 ±0.042	0.962 ±0.020	0.000 ±0.000
TRCE [7]	0.539 ±0.014	0.627 ±0.016	0.000 ±0.000	0.091 ±0.134	0.063 ±0.041	0.852 ±0.024	0.000 ±0.000	0.602 ±0.021	0.615 ±0.019	0.000 ±0.000	0.742 ±0.151	0.034 ±0.013	0.852 ±0.014	0.000 ±0.000
ECPCS-HC [8]	0.492 ±0.021	**0.635** ±0.024	0.000 ±0.000	0.050 ±0.158	0.003 ±0.002	0.887 ±0.026	0.000 ±0.000	0.586 ±0.042	0.609 ±0.027	0.000 ±0.000	0.127 ±0.206	0.011 ±0.011	0.735 ±0.725	0.000 ±0.000
ECPCS-MC [8]	0.510 ±0.041	0.599 ±0.038	0.000 ±0.000	0.012 ±0.025	0.051 ±0.054	0.930 ±0.035	0.000 ±0.000	0.619 ±0.030	0.628 ±0.014	0.000 ±0.000	0.327 ±0.320	0.052 ±0.044	0.847 ±0.026	0.000 ±0.000
CESHL [9]	0.530 ±0.085	0.623 ±0.633	0.000 ±0.000	0.007 ±0.006	0.076 ±0.049	0.899 ±0.096	0.000 ±0.000	0.582 ±0.053	0.602 ±0.030	0.000 ±0.000	0.471 ±0.413	0.108 ±0.127	0.868 ±0.080	0.000 ±0.000
FCE-f	**0.542** ±0.029	0.630 ±0.032	0.000 ±0.000	0.290 ±0.232	0.115 ±0.072	0.951 ±0.030	0.000 ±0.000	0.612 ±0.062	0.623 ±0.073	0.000 ±0.002	0.831 ±0.114	0. 224 ±0.065	0.931 ±0.025	0.002 ±0.005
FCE	0.521 ±0.026	0.603 ±0.022	**0.219** ±0.058	**0.942** ±0.013	**0.534** ±0.078	**0.993** ±0.004	**0.431** ±0.038	0.582 ±0.069	0.593 ±0.083	**0.113** ±0.027	**0.982** ±0.002	**0.613** ±0.084	**0.993** ±0.005	**0.542** ±0.027

TABLE 3: Experimental results with different base clusterings on JAFFE and Yale data sets. The best and second best results are denoted in **bold** and underlined, respectively. FCE-f represents the degenerated version of FCE without the fairness regularized term.

Methods	JAFFE							Yale						
	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE	ACC	NMI	Bal	MNCE	CCE	NE	f_CCE
KM	0.583 ±0.222	0.640 ±0.213	0.150 ±0.264	0.526 ±0.412	0.294 ±0.261	0.940 ±0.052	0.000 ±0.000	0.301 ±0.096	0.320 ±0.129	0.054 ±0.063	0.295 ±0.320	0.131 ±0.097	0.861 ±0.096	0.000 ±0.000
BCE [2]	0.772 ±0.065	0.811 ±0.065	0.000 ±0.000	0.391 ±0.315	0.184 ±0.131	0.953 ±0.026	0.000 ±0.000	0.433 ±0.023	0.484 ±0.032	0.016 ±0.035	0.103 ±0.218	0.073 ±0.045	0.924 ±0.034	0.000 ±0.000
RCE [3]	0.869 ±0.069	0.868 ±0.041	0.000 ±0.000	0.599 ±0.293	0.348 ±0.212	0.980 ±0.017	0.000 ±0.000	0.406 ±0.024	0.452 ±0.016	0.000 ±0.000	0.000 ±0.054	0.024 ±0.033	0.814 ±0.029	0.000 ±0.000
LWGP [4]	0.825 ±0.072	0.863 ±0.039	0.000 ±0.000	0.194 ±0.299	0.151 ±0.227	0.950 ±0.028	0.000 ±0.000	0.421 ±0.041	0.475 ±0.038	0.000 ±0.000	0.000 ±0.000	0.032 ±0.017	0.879 ±0.039	0.000 ±0.000
LWEA [4]	0.830 ±0.073	0.860 ±0.036	0.000 ±0.000	0.431 ±0.302	0.194 ±0.179	0.961 ±0.021	0.000 ±0.000	0.430 ±0.048	0.484 ±0.041	0.000 ±0.000	0.000 ±0.000	0.033 ±0.007	0.888 ±0.042	0.000 ±0.000
DREC [5]	0.815 ±0.089	0.847 ±0.054	0.000 ±0.000	0.354 ±0.369	0.225 ±0.250	0.956 ±0.028	0.000 ±0.000	0.439 ±0.035	0.519 ±0.028	0.006 ±0.021	0.044 ±0.140	0.075 ±0.047	0.933 ±0.016	0.000 ±0.000
RSEC [6]	0.893 ±0.070	0.882 ±0.059	0.000 ±0.000	0.795 ±0.103	0.548 ±0.177	0.991 ±0.008	0.000 ±0.000	0.456 ±0.039	0.527 ±0.036	0.009 ±0.028	0.054 ±0.172	0.198 ±0.107	0.970 ±0.012	0.000 ±0.000
TRCE [7]	0.874 ±0.083	0.883 ±0.054	0.000 ±0.000	0.477 ±0.443	0.403 ±0.364	0.969 ±0.034	0.000 ±0.000	0.427 ±0.036	0.501 ±0.033	0.000 ±0.000	0.000 ±0.000	0.054 ±0.045	0.913 ±0.023	0.000 ±0.000
ECPCS-HC [8]	0.731 ±0.096	0.799 ±0.053	0.000 ±0.000	0.000 ±0.000	0.046 ±0.019	0.903 ±0.015	0.000 ±0.000	0.365 ±0.027	0.414 ±0.034	0.000 ±0.000	0.000 ±0.000	0.016 ±0.002	0.751 ±0.048	0.000 ±0.000
ECPCS-MC [8]	0.801 ±0.096	0.820 ±0.089	0.000 ±0.000	0.232 ±0.338	0.159 ±0.174	0.948 ±0.040	0.000 ±0.000	0.377 ±0.036	0.421 ±0.039	0.016 ±0.035	0.103 ±0.219	0.054 ±0.053	0.883 ±0.039	0.000 ±0.000
CESHL [9]	0.677 ±0.144	0.734 ±0.108	0.000 ±0.000	0.077 ±0.243	0.053 ±0.106	0.850 ±0.091	0.000 ±0.000	0.300 ±0.052	0.326 ±0.072	0.000 ±0.000	0.000 ±0.000	0.012 ±0.003	0.644 ±0.098	0.000 ±0.000
FCE-f	0.908 ±0.061	0.897 ±0.044	0.000 ±0.000	0.844 ±0.077	0.592 ±0.131	0.993 ±0.007	0.000 ±0.000	**0.470** ±0.034	**0.535** ±0.024	0.000 ±0.000	0.000 ±0.000	0.226 ±0.054	0.971 ±0.007	0.000 ±0.000
FCE	**0.955** ±0.051	**0.940** ±0.056	**0.500** ±0.000	**0.988** ±0.002	**0.885** ±0.040	**0.999** ±0.000	**0.658** ±0.014	0.458 ±0.043	0.490 ±0.039	**0.157** ±0.049	**0.742** ±0.124	**0.709** ±0.045	**0.997** ±0.000	**0.490** ±0.089